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Reversible Thinking Ability in Solving Mathematics Problems

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Abstract

Reversible thinking is the process of thinking by reversing the order of actions. Reversible thinking is an important aspect in helping to improve students' problem-solving ability. This research aims to analyze qualitative studies related to students' reversible thinking ability at elementary, middle, and high school levels in the period 2002 - 2022. The research method uses a Systematic Literature Review (SLR) that collects primary data that has been published in Sinta and Scopus indexed journals. Data extraction was adjusted to the selection criteria so that 19 articles were collected. Data analysis used a qualitative approach. Data grouping was done based on publication year, education level, demographics, journal indexer, material analyzed, and research results. The results shows that research related to reversible thinking ability became a trend in research in 2015-2022 with the topic of algebra. The studies related to this reversible thinking process are mostly conducted in the Java region at the elementary school level. However, this reversible thinking ability at all levels of education is still low. Based on the three aspects of reversible thinking, working backwards is the most difficult aspect for students to do. One of the reasons is because students do not understand the concept thoroughly. This is a concern for researchers and educators to conduct research related to students' reversible thinking ability, especially outside the region with topics that are still rarely studied.

Keywords: Extraction Data, Reversible Thinking Ability, Systematic Literature Review, Working Backward

Abstrak

Berpikir reversibel adalah proses berpikir dengan membalik urutan tindakan. Berpikir reversibel merupakan aspek penting dalam membantu meningkatkan kemampuan pemecahan masalah siswa. Penelitian ini bertujuan untuk menganalisis studi kualitatif yang berkaitan dengan kemampuan berpikir reversibel siswa di tingkat sekolah dasar, menengah, dan atas pada periode 2002 - 2022. Metode penelitian menggunakan Systematic Literature Review (SLR) yang mengumpulkan data primer yang telah dipublikasikan di jurnal terindeks Sinta dan Scopus. Ekstraksi data disesuaikan dengan kriteria seleksi sehingga terkumpul 19 artikel. Analisis data menggunakan pendekatan kualitatif. Pengelompokan data dilakukan berdasarkan tahun publikasi, tingkat pendidikan, demografi, pengindeks jurnal, materi yang dianalisis, dan hasil penelitian. Hasil penelitian menunjukkan bahwa penelitian yang berkaitan dengan kemampuan berpikir reversibel menjadi trend dalam penelitian pada tahun 2015-2022 dengan topik aljabar. Penelitian-penelitian yang berkaitan dengan proses berpikir reversibel ini sebagian besar dilakukan di wilayah Jawa pada jenjang sekolah dasar. Namun, kemampuan berpikir reversibel ini pada semua jenjang pendidikan masih rendah. Berdasarkan ketiga aspek berpikir reversibel tersebut, bekerja mundur merupakan aspek yang paling sulit dilakukan siswa. Salah satu penyebabnya adalah karena siswa tidak memahami konsep secara menyeluruh. Hal ini menjadi perhatian bagi para peneliti dan pendidik untuk melakukan penelitian terkait kemampuan berpikir reversibel siswa, terutama di luar daerah dengan topik yang masih jarang diteliti.

Kata kunci: Ekstrasi data, Kemampuan Berpikir Reversibel, Sistematik Literatur Reviu, Bekerja mundur

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INTRODUCTION

Mathematics education has an important role in building quality human resources (Firma, 2010). Therefore, mathematics education is important to be taught to students at all levels. Based on the Regulation of the Minister of Education, Culture, Research and Technology of the Republic of Indonesia No. 7 of 2022 concerning Content Standards, it is stated that learning mathematics is a subject that must be taught starting from the basic education level to the secondary education level

(Kemendikbudristek, 2022). In addition, learning mathematics also builds some basic skills needed by students. NCTM has determined these five basic abilities, namely problem-solving, reasoning, communication, connection, and representation (Kurnia Putri et al., 2019).

Among the five basic skills above, problem-solving skills are always relevant to today's educational and industrial needs. Because that ability is the basis for finding alternative solutions to a problem to achieve goals (G. Polya, 2010). Problem-solving ability is the core of learning mathematics (Maf'ulah & Juniati, 2020b). However, in reality, these competencies are still a scourge for most students, where the level of students' problem-solving abilities is still low at various levels and cases such as in the flat-shaped material in junior high school (Sari & Aripin, 2018), difficulties in completing algebra material at the junior high school level (Marisa et al., 2020), and the low problem-solving ability of high school students on arithmetic material (Pirmanto et al., 2020).

According to Krutetskii and Polya, one aspect of thinking that affects problem-solving abilities is a reversible thinking process (Krutetskii, 1978; G. Polya, 2010)). Reversible thinking is a thought process by reversing the sequence of occurrence. Students must be able to think logically in two reversible ways, making two-way connections between concepts, principles, and procedures to reinforce schemas (Flanders, 2014). There are two reversible thinking processes: a) thinking forward processes that can be identified from 2 aspects of thinking, namely negation thinking – doing the inverse of related operations and reciprocity thinking – doing reciprocal correlations or equivalent relationships in equations; b) backward thinking processes that can be identified from 3 aspects of thinking, namely negation, reciprocity and the ability to think the return to basic points after getting the results (Maf'Ulah et al., 2019). Therefore, reversible thinking skills become an important aspect that must be researched to help improve students' problem-solving abilities.

Research on reversible thinking has attracted the attention of researchers in recent years. Based on searches conducted in national and international journals on reversible thinking skills, the first research was conducted in 1978 by Adi on the conceptual relationship between intellectual development and reversibility in teachers (Adi, 1978). Then it became a hot topic in research from 2016 until now (Maf'ulah et al., 2016; Kurniawati & Sutiarso, 2022).

From various studies that have been carried out in the past few years, it is necessary to have a thorough review of reversible thinking skills in the learning process to observe research trends related to this topic. Therefore, research was conducted in the form of a systematic review of students' reversible abilities using the Systematic Literature Review (SLR) method.

The SLR research method aims to collect and synthesize comprehensive research data that is based on specific questions, organized procedures, transparent and can be replicated at every step in the process (Kek & Huijser, 2011; Juandi & Tamur, 2021). The SLR method allows a more objective assessment of evidence than traditional narrative reviews and can contribute to resolving uncertainties and identifying areas where further study is needed (Egger et al., 2009).

Data collection related to reversible thinking research is summarized based on several questions such as year of publication, level of study, demographics, journal indexers, analytical techniques, materials, and results contained in the article. Through the data obtained, the researcher posed the following related questions: (1) How is the description of research findings related to students' reversible thinking based on the year of publication? (2) How is the description of research findings related to students' reversible thinking based on the level of study? (3) How is the description of research findings related to students' reversible thinking based on the journal index? (4) How is the description of research findings related to students' reversible thinking based on the research location? (5) How is the description of research findings related to students' reversible thinking based on mathematics topics? (6) How is the description of research findings related to students' reversible thinking based on the results contained in the article?

METHOD

Systematic Literatur Review

The method used in this research is a Systematic Literature Review (SLR). Where this method is carried out by synthesizing the results of scientific studies to answer certain research questions in a transparent and reproducible way (Lame, 2019). In this study, a survey was conducted on primary data in the form of basic research results related to students' reversible thinking abilities. The survey treatment includes 4 stages of research, namely: (1) The first stage is observing the structure based on the research context, defining a review protocol, and conducting a search; (2) The second stage removes unnecessary data; (3) The third stage is data extraction by categorizing data items (4) The fourth stage reports the results of the research discussion (Marbawi & Salim, 2019).

Inclusion criteria were intended to obtain the articles that met the research objectives. Exclusion criteria can be used to eliminate irrelevant studies from the selected studies (Zlatko Stapić et al., 2006). The inclusion criteria set out in this study are as follows: (1) The study was conducted in the field of mathematics; (2) The study conducted analyzed the students' reversible ability; (3) The study was conducted on students at the elementary, secondary and higher levels; (4) The study was published in the last 20 years, that from 2002 to 2022; (5) Studies published in journals indexed by Sinta and Scopus; (6) The study must contain the approach or method used.

Population and Sample

The population in this study is all studies on the students' reversible thinking. The studies collected amounted to 22 studies. Furthermore, one article with a quantitative approach, one article not indexed by Scopus or Sinta, and one article with an inappropriate publication year were deleted. Articles were selected based on the inclusion criteria so 19 articles were obtained.

Data Collection Technique

Information collected in the form of primary research on reversible thinking has been published in national journals indexed by Sinta and international journals indexed by Scopus by search engines such as Google Scholar, Semantic Scholar, and ERIC.

Data analysis Technique

Descriptive qualitative data analysis was used in this study.

RESULTS AND DISCUSSION

Studies by Year of Publication

The data presented in Figure 1 below is a study of students' reversible thinking abilities for the last twenty years, starting from 2002 to 2022.

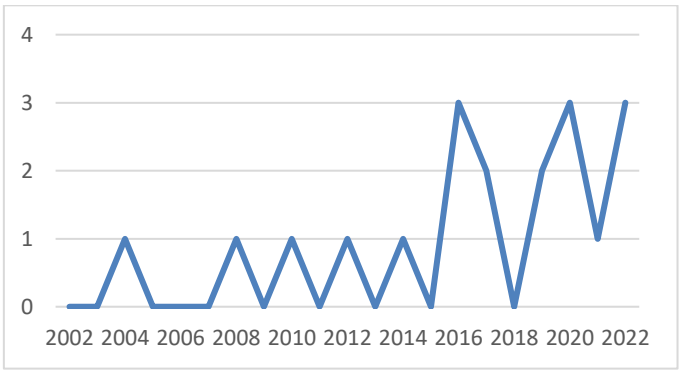

Figure 1. Reversible thinking ability graph by year of publication

Figure 1 shows that studies related to reversible thinking skills in the last few years have become a concern for researchers. In 2015, studies related to reversible thinking processes began to become a research trend. In that year there was a significant increase compared to the previous year. Because there is an urgency of the reversible thinking process on students' problem-solving abilities (Maf'ulah & Juniati, 2020). The problem-solving ability itself is a competency that is still relevant to the development of education and industry. Because problem-solving skills are the basis for determining alternative solutions to problems (G. Polya, 2010). This can be a new object for researchers to develop reversible thinking processes.

Studies by Education Level

Studies related to reversible thinking skills have been investigated at all levels of education, starting from elementary, junior high, high school and college, with the distribution of the data in Figure 2 below:

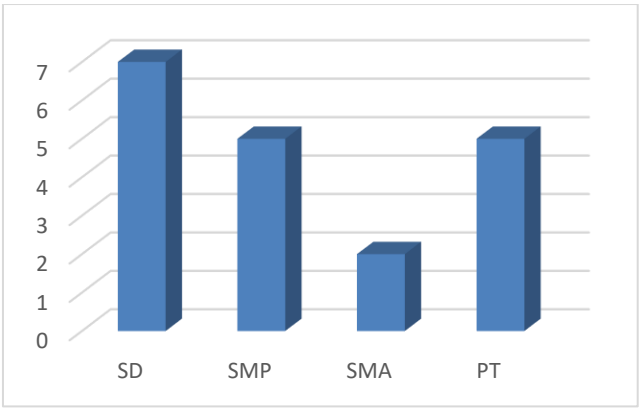

Figure 2. Graph of reversible thinking skills by education level

Based on Figure 2, the study of reversible thinking skills is mostly studied at the elementary school level than the other level of study. One of the reasons why is that children do have difficulty gaining a conceptual understanding of the inverse relationship, especially in the multiplication material that is first taught (Robinson & LeFevre, 2012). However, this does not mean that other levels of study are not important to analyze. Because there are many mathematics topics related to reversible thinking at the high school and university levels such as functions, derivatives, linear equations, mathematical induction, and other materials. This can be a recommendation for researchers because the reversible thinking ability of students at that level is also still low.

Studies Based on Journal Indexers

Figure 3 below visualizes the distribution of studies based on journal indexes starting from journals indexed by Scopus from Q1 to Q4 and indexed by Sinta from Sinta 1 to Sinta 3. Based on data from journals indexed by Scopus, the distribution of data is mostly published in journals with an index of Q1. Meanwhile, nationally, studies related to reversible thinking skills are widely published in journals with an index of Sinta 2 and Sinta 3.

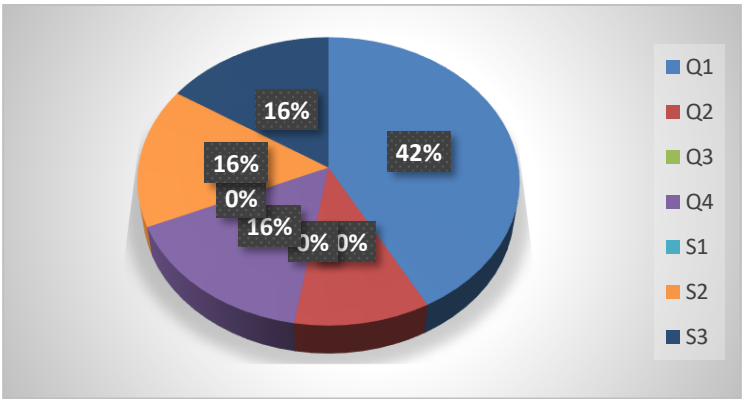

Figure 3. Graph of reversible thinking skills based on journal indexers

Based on the presentation of journal publications above, this provides a great opportunity for authors to be able to publish their articles in Scopus-indexed journals.

Studies By Demographics

Details of the distribution of studies based on demographics will be presented into 2 types, namely the demographic distribution of the Scopus-indexed articles which continents the research on reversible thinking skills is researched on is presented in Figure 4. Then the demographic distribution of the indexed articles in Scopus and Sinta will also be presented whose research is located in areas in Indonesia are presented in Figure 5.

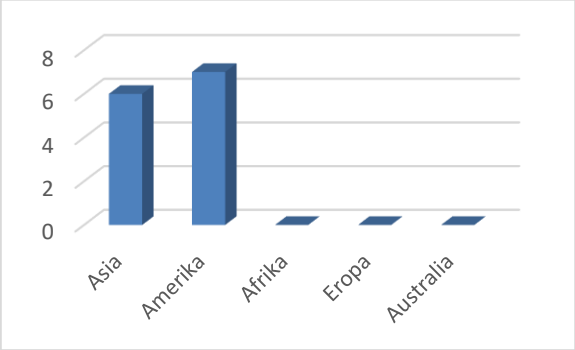

Figure 4. Reversible Thinking Ability Graph Based on Distribution Across Continents

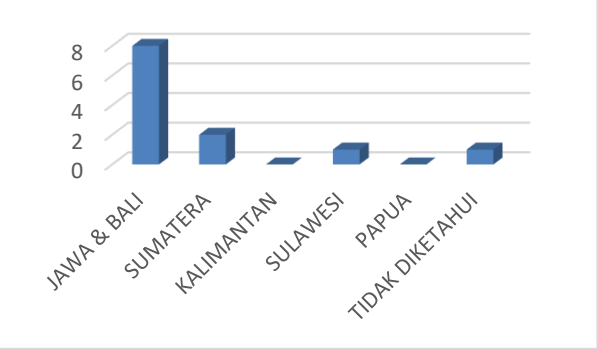

Figure 5. Graph of Reversible Thinking Ability Based on Distribution in Indonesia

Figure 4 shows that studies related to this reversible thinking ability have become a trend in research conducted by researchers in the Americas and Asia. However, studies on this topic in Africa, Europe and Australia have not been found in Scopus-indexed journals. Furthermore, for Figure 5, for the Scopus and Sinta-indexed articles that conduct studies in regions in Indonesia, the distribution is mostly in Java. These results support the research proposed by Arianti & Juandi (2018) that research in eastern Indonesia is still low. The results of this study can be used as an opportunity by researchers to improve the reversible ability of students to conduct related research in eastern Indonesia.

Studies Based on Material

Topics in mathematics are divided into several fields including arithmetic, algebra, geometry, statistics, trigonometry and graph theories. Many of the topics in mathematics require reversible thought processes (Kurniawati & Sutiarso, 2022). Figure 6 below presents the distribution of studies based on mathematical topics:

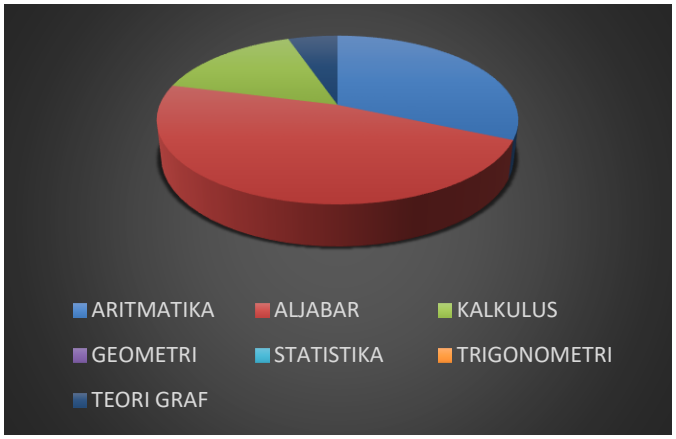

Figure 6. Reversible Thinking Ability Graph Based on Materi

Figure 6 above demonstrates that the topic that is widely used by researchers to measure students' reversible thinking skills is the topic in algebra because this algebra is a supporting material for other mathematics topics. In addition, studying mathematics on the topic of algebra can affect students' multiple intelligences (Ahmad Nizar Rangkuti, 2015). So, it has a high urgency to be taught at the elementary education level. However, in other topics, there are also aspects of reversible thinking that are recommended to be researched such as the topic of inverse functions in calculus topics (Maf'Ulah et al., 2019).

Results-Based Studies

From all studies that measure students' reversible thinking skills, it is said that students' reversible thinking skills are still low at all levels of education, including primary school, secondary school and university (Maf'Ulah et al., 2019; Maf'ulah & Juniati, 2020b; Sutiarso, 2020; Robinson & LeFevre, 2012; Maf'ulah et al., 2016). The low reversible thinking ability of students is identified from the results of tests that require students to think reversibly. In multiple-choice type questions, students do not work backwards but rather by matching answers or by trial and error. Even students who have difficulty do not work at all to find a solution (Balingga et al., 2016; Sangwin & Jones, 2017). This is because students do not build knowledge as a whole. A complete understanding can help students in working on tests with two directions of completion (Flanders, 2014). In addition, another factor that causes students' low reversible thinking skills is because students are unable to make connections between concepts, procedures and principles. This makes it difficult for students to solve questions by thinking backwards.

In addition to analysing students' ability to think reversibly, the results of other studies identify aspects of thinking that occur in students in the reversible thinking process, namely aspects of negation, reciprocity, and the ability to return to the initial data after obtaining results, which are described in research conducted by Sapawardi and Mafulah (Saparwadi et al., 2020; Maf'ulah et al., 2017). The negation aspect is identified when students perform the process of moving the variable part (number) which produces the identity element. Meanwhile, the reciprocal aspect is identified when students can

perform equivalent operations on both segments in an equation. Meanwhile, the most difficult aspect of thinking done by students is when students are required to be able to work backwards after obtaining results to return to the starting point. This third aspect is very difficult because, in addition to students being required to be able to perform operations correctly, students must also understand the concept as a whole to be able to work in reverse sequentially. But even though it is difficult, elementary school students who have won the national science olympiad are able to do the reverse thinking aspect well.

On the other hand, the results of the study also show that reversible mental activities that occur optimally will have an impact on optimal problem-solving abilities (Maf'ulah & Juniati, 2020b; Ramful, 2014; Robinson & LeFevre, 2012). Because when students can carry out the process of working forward and working backward in a balanced manner mental activity can occur optimally which has an impact on optimizing problem-solving skills. This indicates that there is a significant relationship between reversible thinking ability and problem-solving ability. Another study also identified aspects of reversible thinking in terms of students' optimistic, choleric and phlegmatic personalities (Hackenberg, 2010). All three personalities can work on problems that demand reversible thinking skills in ways that are unique to each personality.

CONCLUSION

In recent years, studies related to reversible thinking skills have become a trend for research because of the urgency of reversible thinking processes for students in the problem-solving process. Compared to other levels of education, the study of reversible thinking processes is predominantly conducted in primary schools with algebra material because the material is a support for other mathematics topics, so it has a high urgency to be taught at the basic education level. Studies with Scopus-indexed publications are mostly carried out in countries in America. As for Indonesia itself, most of the studies were conducted in Java & Bali, and then Sumatra. In addition, it was also found that the reversible thinking ability of students at all levels is still low, it is because students do not have the concept as a whole, so it is difficult to solve by working backward. This could be a concern for researchers to develop effective learning designs in improving reversible thinking ability. Based on the importance of reversible thinking skills, it is recommended for educators or researchers to conduct research related to students' reversible thinking skills at the secondary school level outside Java on calculus, trigonometry, and statistics material because these topics remain underexplored.

REFERENCES

Adi, H. (1978). Development and Intellectual of Thought in Equation. *Journal for Research in Mathematics Education*, *9*(3), 204–213.

Ahmad Nizar Rangkuti. (2015). Hubungan Kecerdasan Ganda Dengan Hasil Belajar Topik Aljabar Siswa Kelas VIII Mtsn 2 Padangsidimpuan. *Logaritma*, *III*(02), 1–11.

Ariati, C., & Juandi, D. (2018). Kemampuan Penalaran Matematis. *Angewandte Chemie International*

Edition, 6(11), 951–952., *1*(2), 10–27.

Balingga, E., Prahmana, R. C. I., & Murniati, N. (2016). Analisis Kemampuan Reversibilitas Siswa MTs Kelas VII dalam Menyusun Persamaan Linier. *Jurnal Review Pembelajaran Matematika*, *1*(2), 117–131. https://doi.org/10.15642/jrpm.2016.1.2.117-131

Egger, M., Davey Smith, G., & Sterne, J. (2009). 623 Systematic reviews and meta-analysis. In R. Detels, R. Beaglehole, M. A. Lansang, & M. Gulliford (Eds.), *Oxford Textbook of Public Health* (p. 0). Oxford University Press. https://doi.org/10.1093/med/9780199218707.003.0039

Firma, Y. (2010). Jurnal pendidikan matematika. *Jurnal Pendidikan Matematika*, *4*(2), 111–121. https://ejournal.unsri.ac.id/index.php/jpm/article/view/816/229

Flanders. (2014). Investigating Flexibility , Reversibility , And Multiple Representations In A Calculus Environment by Steven Todd Flanders Bachelor of Science , Grove City College , 2004 Master of Education , Gannon University , 2006 Submitted to the Graduate Faculty of. *Thesis*.

G. Polya. (2010). How to Solve it. In *Stochastic Optimization in Continuous Time*. https://doi.org/10.1017/cbo9780511616747.007

Hackenberg, A. J. (2010). Students' reasoning with reversible multiplicative relationships. *Cognition and Instruction*, *28*(4), 383–432. https://doi.org/10.1080/07370008.2010.511565

Juandi, D., & Tamur, M. (2021). The impact of problem-based learning toward enhancing mathematical thinking: A meta-analysis study. *Journal of Engineering Science and Technology*, *16*(4), 3548–3561.

Kek, M.Y.C.A.; and Huijser, H. (2011). The power of problem-based learning in developing critical thinking skills: Preparing students for tomorrow's digital futures in today's classrooms. Higher Education Research and Development, 30(2011), 329-341. https://doi.org/10.1080/07294360.2010.501074

Kemendikbudristek. (2022). *Permendikbud Tentang Standar Isi Pada PAUD, Jenjang Pendidikan Dasar dan Menengah*. 1–122.

Krutetskii, V. A. (1976). The psychology of mathematical abilities in schoolchildren. Chicago: The University of Chicago Press.

Kurnia Putri, D., Sulianto, J., & Azizah, M. (2019). Kemampuan Penalaran Matematis Ditinjau dari Kemampuan Pemecahan Masalah. *International Journal of Elementary Education*, *3*(3), 351. https://doi.org/10.23887/ijee.v3i3.19497

Kurniawati, D., & Sutiarso, S. (2022). Analisis Kemampuan Reversible Thinking Matematis Siswa SMA Pada Konsep Kalkulus. *... Cendekia: Jurnal Pendidikan Matematika*. https://j-cup.org/index.php/cendekia/article/view/1464

Lame, G. (2019). Systematic literature reviews: An introduction. *Proceedings of the International Conference on Engineering Design, ICED, 2019-Augus*(July), 1633–1642. https://doi.org/10.1017/dsi.2019.169

Maf'Ulah, S., Fitriyani, H., Yudianto, E., Fiantika, F. R., & Hariastuti, R. M. (2019). Identifying the

reversible thinking skill of students in solving function problems. *Journal of Physics: Conference Series*, *1188*(1). https://doi.org/10.1088/1742-6596/1188/1/012033

Maf'ulah, S., & Juniati, D. (2020a). Exploring reversible thinking of preservice mathematics teacher students through problem-solving task in algebra. *Journal of Physics: Conference Series*, *1663*(1). https://doi.org/10.1088/1742-6596/1663/1/012003

Maf'ulah, S., & Juniati, D. (2020b). The Effect of Learning with Reversible Problem-Solving Approach on Prospective-Math-Teacher Students' Reversible Thinking. *International Journal of Instruction*. https://eric.ed.gov/?id=EJ1249071

Maf'ulah, S., Juniati, D., & Siswono, T. (2016). Pupils' Error on the Concept of Reversibility in Solving Arithmetic Problems. *Educational Research and Reviews*, *11*(18), 1775–1784. https://doi.org/10.5897/ERR2016.2895

Maf'ulah, S., Juniati, D., & Siswono, T. Y. E. (2017). The aspects of reversible thinking in solving algebraic problems by an elementary student winning national Olympiad medals in science. *World Transactions on Engineering and Technology Education*, *15*(2), 189–194.

Marbawi, M. I., & Salim, T. A. (2019). Mempertahankan keaslian arsip elektronik di era digital berdasarkan tinjauan literatur sistematis. *Berkala Ilmu Perpustakaan Dan Informasi*, *15*(2), 149. https://doi.org/10.22146/bip.47370

Marisa, G., Syaiful, S., & Hariyadi, B. (2020). Analisis Kesalahan Siswa Dalam Menyelesaikan Soal Operasi Aljabar Berdasarkan Taksonomi SOLO. *Jurnal Pendidikan Matematika*, *11*(1), 77. https://doi.org/10.36709/jpm.v11i1.9971

Pirmanto, Y., Farid Anwar, M., & Bernard, M. (2020). Analisis Kesulitan Siswa SMA dalam Menyelesaikan Soal Pemecahan Masalah pada Materi Barisan dan Deret dengan Langkan-langkah Menurut Polya. *Jurnal Pembelajaran Matematika Inovatif*, *3*(4), 371–384. https://doi.org/10.22460/jpmi.v3i4.371-384

Ramful, A. (2014). Reversible reasoning in fractional situations: Theorems-in-action and constraints. *The Journal of Mathematical Behavior*. https://www.sciencedirect.com/science/article/pii/S0732312313000990

Robinson, K. M., & LeFevre, J. A. (2012). The inverse relation between multiplication and division: Concepts, procedures, and a cognitive framework. *Educational Studies in Mathematics*, *79*(3), 409–428. https://doi.org/10.1007/s10649-011-9330-5

Sangwin, C. J., & Jones, I. (2017). Asymmetry in student achievement on multiple-choice and constructed-response items in reversible mathematics processes. *Educational Studies in Mathematics*, *94*(2), 205–222. https://doi.org/10.1007/s10649-016-9725-4

Saparwadi, L., Sa'dijah, C., As'ari, A. R., & Chandrad, T. D. (2020). The aspects and stages of reversible thinking of secondary school students in resolving the problems of fractional numbers. *Systematic Reviews in Pharmacy*, *11*(6), 1302–1310. https://doi.org/10.31838/srp.2020.6.190

Sari, A. R., & Aripin, U. (2018). Analisis Kesalahan Siswa Dalam Menyelesaikan Soal Cerita Bangun Datar Segiempat Ditinjau Dari Kemampuan Pemecahan Masalah Matematik Untuk Siswa Kelas Vii. *JPMI (Jurnal Pembelajaran Matematika Inovatif)*, *1*(6), 1135. https://doi.org/10.22460/jpmi.v1i6.p1135-1142

Sutiarso, S. (2020). Analysis of Student Reversible Thinking Skills on Graph Concept. *Indonesian Journal of Science and Mathematics …*, *3*(2), 185–195. http://103.88.229.8/index.php/IJSME/article/view/6768

Zlatko Stapić, Eva García López, Antonio García Cabot Luis, & Vjeran Strahonja. (2006). Performing systematic literature reviews in software engineering. *Proceedings - International Conference on Software Engineering*, *2006*(December 2014), 1051–1052. https://doi.org/10.1145/1134285.1134500